

\bar{D}^*D and \bar{B}^*B (1^{++}) molecules at N2LO from QSSR*

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Abstract

We use QCD spectral sum rules (QSSR) and the factorization properties of molecule currents to estimate the masses and couplings of the \bar{D}^*D and \bar{B}^*B (1^{++}) molecules at N2LO of PT QCD. We include in the OPE the contributions of non-perturbative condensates up to dimension-eight. With the Laplace sum rules approach (LSR) and in the \overline{MS} -scheme, we obtain $M_{D^*D} = 3738(152)$ MeV, which agrees within the errors with the newly discovered $Z_c(3900)$. For the bottom channel, we find $M_{B^*B} = 10687(232)$ MeV in good agreement with the observed $Z_b(10610)$. Couplings of these states to the currents are also extracted. Our results are improvements of the LO ones in the existing literature.

Keywords: QCD Spectral Sum Rules, molecule states, heavy quarkonia.

1. Introduction

The recent discovery of the $Z_c(3900)$ 1^{++} by Belle [1] and BESIII [2] from its $J/\psi\pi^\pm$ decays has motivated different theoretical analysis [3]. However, all of the previous analysis like e.g. in [4] from QCD Spectral Sum Rules (QSSR) [5, 6] have been done at LO of PT QCD. In this paper, we are going to use QSSR to evaluate the mass and coupling of the 1^{++} \bar{D}^*D and \bar{B}^*B molecules at N2LO in the PT series and compare the results with those obtained at lowest order and with experiments.

2. QCD analysis of spin one molecule

• Current and two-point fonction

The current for this molecule state is given by:

$$\begin{aligned} J^\mu &\equiv (\bar{Q}\gamma^\mu q)(\bar{q}\gamma^5 Q), \\ Q &\equiv c, b \text{ and } q \equiv u, d. \end{aligned} \quad (1)$$

The associated two-point correlation function is:

$$\begin{aligned} \Pi_{mol}^{\mu\nu}(q) &= i \int d^4x e^{iq \cdot x} \langle 0 | T J^\mu(x) J^{\nu\dagger}(0) | 0 \rangle \\ &= -(q^2 g^{\mu\nu} - q^\mu q^\nu) \Pi_{mol}(q^2) \\ &\quad + q^\mu q^\nu \Pi_{mol}^{(0)}(q^2), \end{aligned} \quad (2)$$

where Π_{mol} and $\Pi_{mol}^{(0)}$ are respectively associated to the spin 1 and 0 molecule states. In the QSSR method and parametrizing the spectral function by one resonance plus a QCD continuum, the lowest resonance mass M_H and coupling f_H normalized as:

$$\langle 0 | J^\mu | H \rangle = f_H M_H^4 \epsilon^\mu, \quad (3)$$

can be extracted by using the Laplace sum rules (LSR) which gives two well-known sum rules [6]:

$$M_H^2 = \frac{\int_{4m_Q^2}^{t_c} dt t e^{-t\tau} \frac{1}{\pi} \text{Im} \Pi_{mol}^{OPE}(t)}{\int_{4m_Q^2}^{t_c} dt e^{-t\tau} \frac{1}{\pi} \text{Im} \Pi_{mol}^{OPE}(t)} \quad (4)$$

and

$$f_H^2 = \frac{\int_{4m_Q^2}^{t_c} dt e^{-t\tau} \frac{1}{\pi} \text{Im} \Pi_{mol}^{OPE}(t)}{e^{-\tau M_H^2} M_H^8} \quad (5)$$

where m_Q is the heavy quark mass, τ the sum rule parameter and t_c the continuum threshold.

• The QCD two-point function at N2LO

To derive the results at N2LO, we assume factorization and then use the fact that the two-point function of molecule state can be written as a convolution of the spectral functions associated to quark bilinear currents.

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In the spin one case, we have [7, 8]:

$$\frac{1}{\pi} \text{Im}\Pi_{\text{mol}}^{(1)}(t) = \theta(t - 4M_Q^2) \left(\frac{1}{4\pi} \right)^2 t^2 \int_{M_Q^2}^{\sqrt{t}-M_Q} dt_1 \times \int_{M_Q^2}^{\sqrt{t}-\sqrt{t_1}} dt_2 \lambda^{3/2} \frac{1}{\pi} \text{Im}\Pi^{(1)}(t_1) \frac{1}{\pi} \text{Im}\Pi^{(0)}(t_2) \quad (6)$$

with the phase space factor:

$$\lambda = \left(1 - \frac{(\sqrt{t} - \sqrt{t_1})^2}{t} \right) \left(1 - \frac{(\sqrt{t_1} + \sqrt{t_2})^2}{t} \right). \quad (7)$$

$\text{Im}\Pi^{(1)}(t)$ and $\text{Im}\Pi^{(0)}(t)$ are respectively the spectral function associated to the vector and to the pseudoscalar bilinear currents. The QCD expression of the spectral functions for bilinear currents are already known up to order α_s^2 and including non-perturbative condensates up to dimension 6. It can be found in [9–12] for the on-shell mass M_Q . We shall use the relation between the on-shell M_Q and the running mass $\bar{m}_Q(\nu)$ to transform the spectral function into the \overline{MS} -scheme [13, 14]:

$$M_Q = \bar{m}_Q(\nu) \left[1 + \frac{4}{3} a_s + (16.2163 - 1.0414 n_l) a_s^2 + \ln \left(\frac{\nu}{M_Q} \right)^2 (a_s + (8.8472 - 0.3611 n_l) a_s^2) + \ln^2 \left(\frac{\nu}{M_Q} \right)^2 (1.7917 - 0.0833 n_l) a_s^2 \right], \quad (8)$$

where $n_l = n_f - 1$ is the number of light flavours and $a_s(\nu) = \alpha_s(\nu)/\pi$ at the scale ν .

• QCD parameters

The PT QCD parameters which appear in this analysis are α_s , the charm and bottom quark masses $m_{c,b}$ (the light quark masses have been neglected). We also consider non-perturbative condensates from [15] up to dimension 8 which are the quark condensate $\langle \bar{q}q \rangle$, the two-gluon condensate $\langle g^2 G^2 \rangle$, the mixed condensate $\langle g \bar{q} G q \rangle$, the four-quark condensate $\rho \langle \bar{q}q \rangle^2$, the three-gluon condensate $\langle g^3 G^3 \rangle$, and the two-quark multiply two-gluon condensate $\rho \langle \bar{q}q \rangle \langle g^2 G^2 \rangle$ where ρ indicates the deviation from the four-quark vacuum saturation. Their values are given in Table 1. For the condensates, we shall use these expressions:

$$\langle \bar{q}q \rangle(\nu) = -\hat{\mu}_q^3 \left(\text{Log} \frac{\nu}{\Lambda} \right)^{-2/\beta_1} \quad (9)$$

$$\langle g \bar{q} G q \rangle(\nu) = -M_0^2 \hat{\mu}_q^3 \left(\text{Log} \frac{\nu}{\Lambda} \right)^{-1/3\beta_1} \quad (10)$$

where $\beta_1 = -(1/2)(11 - 2n_f/3)$ is the first coefficient of the β function, $\hat{\mu}_q$ the renormalization group invariant condensate and Λ is the QCD scale.

Parameters	Values.
$\alpha_s(M_\tau)$	0.325(8)
$\Lambda(n_f = 4)$	$(324 \pm 15) \text{ MeV}$
$\Lambda(n_f = 5)$	$(194 \pm 10) \text{ MeV}$
$\bar{m}_c(m_c)$	$(1261 \pm 24) \text{ MeV}$
$\bar{m}_b(m_b)$	$(4177 \pm 22) \text{ MeV}$
$\hat{\mu}_q$	$(263 \pm 7) \text{ MeV}$
M_0^2	$(0.8 \pm 0.2) \text{ GeV}^2$
$\langle \alpha_s G^2 \rangle$	$(7 \pm 2) \times 10^{-2} \text{ GeV}^4$
$\langle g^3 G^3 \rangle$	$(8.2 \pm 2.0) \text{ GeV}^2 \times \langle \alpha_s G^2 \rangle$
$\rho = \langle \bar{q}q \bar{q}q \rangle / \langle \bar{q}q \rangle^2$	(2 ± 1)

Table 1: QCD input parameters (see e.g. [6, 16] and references therein).

3. Mass of the $\bar{D}^* D(1^{++})$ molecule

• τ and t_c stabilities

We study the behavior of the mass in term of LSR variable τ at different values of t_c as shown in Fig.1. We consider as final and conservative result the one corresponding to the beginning of the τ stability for $t_c=18 \text{ GeV}^2$ until the one where t_c stability is reached for $t_c \approx 25 \text{ GeV}^2$.

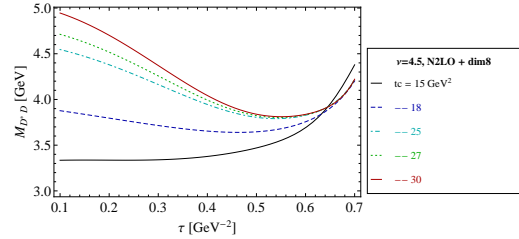


Figure 1: τ -behavior of M_{D^*D} at N2LO for different values of t_c and for $\nu=4.5 \text{ GeV}$

• Convergence of the PT series

According to these analysis, we can notice that the τ -stability begins at $t_c=18 \text{ GeV}^2$ and the t_c -stability is reached from $t_c = 25 \text{ GeV}^2$. Using these two extremal values of t_c , we study in Fig. 2 the convergence of the PT series for a given value of $\nu = 4.5 \text{ GeV}$. We observe that from LO to NLO the mass increases by about +3.5% while from NLO to N2LO, it only increases by +0.5%. This result indicates a good convergence of PT series which validates the LO result obtained in the literature when the running quark mass is used [4].

• ν -stability

We improve our previous results by using different values of ν (Fig. 3). Using the fact that the final result must be independent of the arbitrary parameter ν , we consider as an optimal result the one at the inflexion point

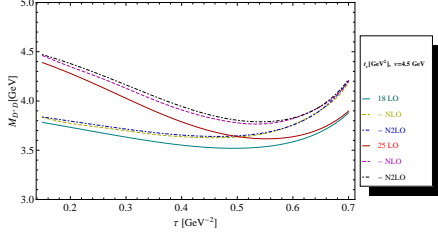


Figure 2: τ -behavior of M_{D^*D} for different values of $t_c=18$ and 25 GeV^2 and $\nu=4.5 \text{ GeV}$ and for different truncation of the PT series

for $\nu \simeq (4.0 - 4.5) \text{ GeV}$:

$$M_{D^*D} = 3738(150)(23) \text{ MeV} , \quad (11)$$

where the second error comes from the localisation of the inflexion point. This result agrees within the errors with the observed $Z_c(3900)$ candidate.

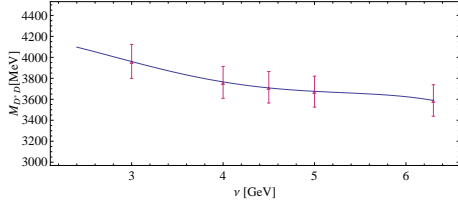


Figure 3: ν -behaviour of M_{D^*D} at N2LO

4. Coupling of the $\bar{D}^*D(1^{++})$ molecule

We can do the same analysis to derive the decay constant f_H defined in Eq. (3). Noting that the bilinear pseudoscalar heavy-light current acquires an anomalous dimension, then the decay constant runs as:

$$f_H(\nu) = \hat{f}_H \left(\text{Log} \frac{\nu}{\Lambda} \right)^{2/-\beta_1} , \quad (12)$$

where \hat{f}_H is a scale invariant coupling. Taking the Laplace transform of the correlator, this definition will lead us to the expression of the running coupling in Eq. (5). We show in Fig. 4 the τ -behaviour of the running coupling $f_{D^*D}(\nu)$ for two extremal values of t_c where τ and t_c stabilities are reached. These values are the same as in the mass determination. One can see in this figure that the α_s corrections to the LO term of PT series are still small though bigger than in the case of the mass determination from the ratio of sum rules. It is about +5.13% from LO to NLO and +4.45% from NLO to N2LO. In the Fig. 5, we show the ν behaviour of the invariant coupling \hat{f}_{D^*D} . Taking the optimal result at the minimum for $\nu \simeq 4 \text{ GeV}$, we obtain in units of MeV:

$$\begin{aligned} \hat{f}_{D^*D} &= (7.43 \pm 1.40) \times 10^{-2} \text{ MeV} \Rightarrow \\ f_{D^*D}(\nu) &= (11.57 \pm 2.17) \times 10^{-2} \text{ MeV} , \end{aligned} \quad (13)$$

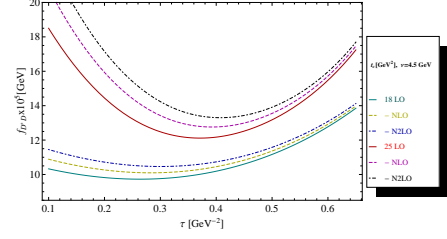


Figure 4: τ -behavior of the running coupling f_{D^*D} for $\nu = 4.5 \text{ GeV}$ and for two extremal values of $t_c = 18$ and 25 GeV^2 .

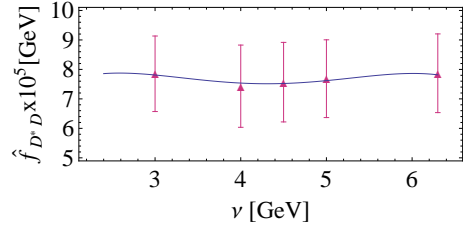


Figure 5: ν -behavior of f_{D^*D} at N2LO

which is comparable with the LO result [17]: $f_{X_c} = (6.5 \pm 1.1) \times 10^{-2} \text{ MeV}$ appropriately normalized of the $X(3872)$.

5. Mass and coupling of the $\bar{B}^*B(1^{++})$ molecule

We do the same analysis in the case of bottom channel. Fig. 6 shows the τ -behavior of mass for $\nu = \bar{m}_b(m_b)$ and Fig. 7 shows its variation versus ν . We have chosen two values of t_c which correspond to the beginning of the τ -stability ($t_c = 120 \text{ GeV}^2$) and to the beginning of t_c stability ($t_c=150 \text{ GeV}^2$). We observe a good convergence of PT series (increase of about 0.46% from LO to NLO and of about 0.35% from NLO to N2LO. Con-

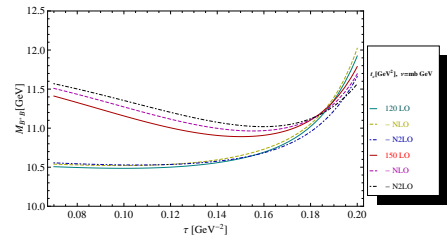


Figure 6: τ -behavior of M_{B^*B} with different values of t_c for $\nu = \bar{m}_b(m_b)$ and for different truncation of the PT series

sidering the one at the minimum in $\nu = \bar{m}_b(m_b)$ as the optimal result, we can deduce:

$$M_{B^*B} = 10687(232) \text{ MeV} , \quad (14)$$

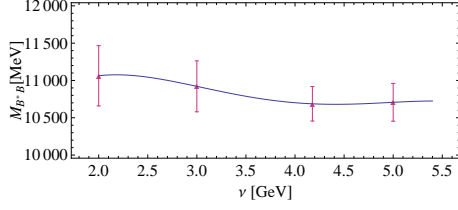


Figure 7: ν -behavior of M_{B^*B} mass at N2LO

where one can notice a good agreement with the observed $Z_b(10610)$ experimental candidate. We show in the Fig. 8 and Fig. 9 the τ and ν -behavior of the coupling for \bar{B}^*B . Like in the case of the charm channel, we will also have the same t_c as in the determination of the mass. Radiative corrections are more important

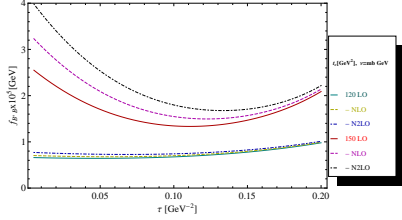


Figure 8: τ -behavior of the running f_{B^*B} coupling for $\nu = \bar{m}_b(m_b)$, $t_c = 120$ and 160 GeV^2 and for different truncations of the PT series.

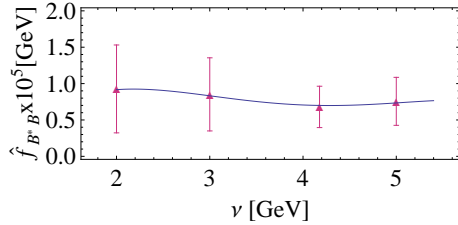


Figure 9: ν -behavior of the invariant coupling \hat{f}_{B^*B} at N2LO.

here than in the case of ratio of moments as expected while the series is slowly convergent. From LO to NLO one has an increase of 10.1% and from NLO to N2LO an increase of about 9.4%. The optimal result for the coupling is obtained at the minimum for $\nu = \bar{m}_b(m_b)$:

$$\hat{f}_{B^*B} = (0.69 \pm 0.29) \times 10^{-2} \text{ MeV} \implies f_{B^*B}(\nu) = (1.22 \pm 0.51) \times 10^{-2} \text{ MeV}, \quad (15)$$

again comparable with the LO result [17]: $f_{X_b} \simeq 10^{-2} \text{ MeV}$ of the X_b predicted at 10144(107) MeV.

6. Conclusions

We have presented improved predictions of QSSR for the masses and couplings of the D^*D and B^*B molecule

states at N2LO of PT series and including up to dimension 8 non-perturbative condensates. Our results given in Eqs. (11) and (14) for the masses are in good agreement within the errors with the experimental candidates $Z_c(3900)$ and $Z_b(10610)$ suggesting that these new states may have large molecule components in their wave functions. However, if one extrapolate the result of Ref. [8] for $\bar{B} - B$ mixing, where the breaking of the four-quark factorization is small (about 10% which should be explicitly checked), one cannot exclude the four-quark assignment for these states. The couplings of these states to the corresponding interpolating currents are given in Eqs. (13) and (15) and are comparable with the ones of the $X_c(3872)$ and $X_b(10144)$ predicted in [17]. The extension of our analysis to some other molecule states is in progress.

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